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Magnetopolarons in quantum dots: comparison of polaronic effects from three to quasi-zero dimensions

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Abstract. The interaction of electrons, confined in a quasi-zero-dimensional quantum dot and longitudinal-optical (LO) phonons, placed in a perpendicular magnetic field, is studied within second-order perturbation theory. Analytic and numerical results are presented for the polaron correction to the Landau levels and the polaron cyclotron mass. It is shown that whether cyclotron resonance results in a resonant magnetopolaron or not depends on the ratio of the confinement energy and the LO phonon energy. The polaronic effects in a magnetic field are compared for all dimensionalities from three to quasi-zero, realizable in the experiment. It is found that these effects increase with reducing the dimensionality. Special attention is directed to the weak-magnetic-field limit and the limit of a weak confinement potential.

1. Introduction

In recent years there has been considerable effort to understand the energy spectrum and the collective excitations of the quasi-two-dimensional (Q2D) electron gas in semiconductor heterostructures, quantum wells and superlattices. Through advances in high-resolution submicrometre lithography the fabrication of semiconductor nanostructures in which quantum confinement of the electronic motion in narrow quantum-well wires (QWWs) and quantum dots (QDs) is realizable. By now the realization of this additional lateral electron confinement with widths below 100 nm for metal-oxide-semiconductor (MOS) structures on Si and III–V compound structures on heterojunctions of GaAs and MOS structures on InSb is well established [1, 2]. Since such widths are in the order of the de Broglie wavelength and are less than the mean free path of the electrons at low temperatures, the physical properties of the electron systems in these QWWs and QDs exhibit quasi-one-dimensional (Q1D) and quasi-zero-dimensional (Q0D) behaviour. With the Q1D and Q0D systems, electron gases of all four dimensionalities from three-dimensional (3D) to Q0D are artificially realized by technological means.

The energy levels of an electron in a strong magnetic field are quantized into Landau levels. If the electron is in a polar semiconductor it also interacts with the optical phonons.

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Hence, the Landau levels are modified by polaronic effects in the following manner: (i) they are shifted to lower energy; (ii) the slopes of the Landau level energies versus magnetic field are changed because of the mass renormalization of the electron; (iii) the Landau levels do not cross the energy level formed by the lowest Landau level plus one optical phonon, because of von Neumann's anticrossing principle; and (iv) the Landau levels are pinned to the energy of that virtual level in high magnetic fields.

The polaron mass is usually experimentally determined by cyclotron resonance. In such an experiment the separation of adjacent Landau levels is measured as a function of the magnetic field B. Hence, in polar semiconductors the cyclotron resonance frequency $\omega_c^* = eB/m_c^*$, with m_c^* the polaron cyclotron mass, is affected by the interaction of the electrons with the optical phonons. For the 3D [3–7] and Q2D [6–9] polarons, considerable work has been done. Two different situations are commonly distinguished in 3D and Q2D systems: the non-resonant magnetopolaron in low magnetic fields and the resonant magnetopolaron in quantizing magnetic fields when the cyclotron energy is approximately equal to the optical phonon energy, but in Q1D systems whether a resonant case is possible or not depends on the confinement potential [10].

2. Theory

In this paper we investigate magnetopolarons in QDs with an isotropic parabolic confinement potential in the two lateral directions. Because the electron is confined within the QD, the magnetopolaron is a bound magnetopolaron. The electron–phonon correction will be calculated within second-order perturbation theory for arbitrary magnetic fields. The unperturbed system, a single electron confined in a zero thickness x–y plane along the z direction at $z = 0$ and confined in a lateral parabolic quantum well potential in the x–y plane in the presence of a quantizing perpendicular magnetic field $\boldsymbol{B} = (0, 0, B)$ neglecting the Zeeman spin-splitting, is described by the Hamiltonian

$$H_e = (1/2m_e)(\boldsymbol{p} + e\boldsymbol{A})^2 + V(\boldsymbol{x}) \tag{1}$$

with m_e the effective conduction band-edge mass, $\boldsymbol{A}$ the vector potential with $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$ and $V(\boldsymbol{x}) = V(x, y) + V(z)$ the confining potential. The lateral parabolic quantum-well potential in the x–y plane is given by $V(x, y) = \frac{1}{2}m_e\Omega^2(x^2 + y^2)$. For the vector potential $\boldsymbol{A}$ we use the symmetric gauge $\boldsymbol{A} = \frac{1}{2}(-y, x, 0)B$. The eigenenergies of the unperturbed Hamiltonian H_e are given by [11]

$$\mathcal{E}_{N_r,m} = \hbar\tilde{\omega}_c(2N_r + |m| + 1) + \tfrac{1}{2}\hbar\omega_c m \qquad N_r = 0, 1, 2, \ldots \qquad m = 0, \pm 1, \pm 2, \ldots \tag{2}$$

where $\tilde{\omega}_c = [(\frac{1}{2}\omega_c)^2 + \Omega^2]^{1/2}$ is the hybrid frequency with $\omega_c = eB/m_e$ the cyclotron frequency, and N_r is the radial and m the azimuthal quantum number, respectively. According to the symmetry properties of the Hamiltonian H_e, the single-particle wave function using cylindrical coordinates in the x–y plane reads [11]

$$\langle \boldsymbol{x}|N_r, m\rangle = \Psi_{N_r m}(\boldsymbol{x}) = C_{N_r m} \exp(im\varphi) r^{|m|} \exp[-(m_e\tilde{\omega}_c/2\hbar)r^2] \times \boldsymbol{F}[-N_r, |m| + 1, (m_e\tilde{\omega}_c/\hbar)r^2]\varphi(z) \tag{3}$$

where $\boldsymbol{F}(a, b, x)$ is the confluent hypergeometric function and

$$C_{N_r m} = [(m_e\tilde{\omega}_c/\hbar)^{(|m|+1)/2}/|m|!][(N_r + |m|)!/\pi N_r!]^{1/2}. \tag{4}$$

According to the strict confinement assumed in the z direction, $|\varphi(z)|^2 = \delta(z)$ is valid. From the calculation of the matrix elements $\langle N_r', m'|\boldsymbol{x}_\parallel|N_r, m\rangle$ with $\boldsymbol{x}_\parallel$ the position vector in the x–y plane, one finds that the dipole-allowed single-electron transitions for a cyclotron resonance experiment have transition energies $\Delta\mathcal{E}_\pm = \hbar\tilde{\omega}_c \pm \frac{1}{2}\hbar\omega_c$.

Our interest is directed to QDs generated by nanostructured gate electrodes via the field effect. Hence, the optical phonons interacting with the electrons are these of the original layered semiconductor structure. Neglecting the effects of interface phonons [12, 13] the Hamiltonian of the electron–phonon interaction including only 3D bulk longitudinal-optical (LO) phonons reads [14]

$$H_{ep} = \left(\frac{4\pi\alpha r_p(\hbar\omega_L)^2}{V_G}\right)^{1/2} \sum_{\boldsymbol{q}} \exp(\mathrm{i}\boldsymbol{q}\boldsymbol{x}) \frac{1}{|\boldsymbol{q}|}[a_L(\boldsymbol{q}) + a_L^+(-\boldsymbol{q})] \tag{5}$$

with $\alpha = \frac{1}{2}(e^2/4\pi\varepsilon_0 r_p)(1/\varepsilon_\infty - 1/\varepsilon_s)/\hbar\omega_L$ the dimensionless 3D polaron coupling constant, $r_p = (\hbar/2m_e\omega_L)^{1/2}$ the corresponding 3D polaron radius, ω_L the frequency of the LO phonons and ε_∞ and ε_s the high-frequency (optical) and the static dielectric constant, respectively, of the semiconductor containing the Q0D confined electrons. $a_L(\boldsymbol{q})$ and $a_L^+(\boldsymbol{q})$ are the phonon destruction and creation operators, respectively, $\boldsymbol{q} = (q_x, q_y, q_z)$ is the 3D wave vector of the 3D bulk LO phonon and V_G is the volume of the sample. The magnetopolaron Hamiltonian is given by

$$H = H_e + \sum_{\boldsymbol{q}} \hbar\omega_L(a_L^+(\boldsymbol{q})a_L(\boldsymbol{q}) + \tfrac{1}{2}) + H_{ep} = H_0 + H_1. \tag{6}$$

The first two terms represent the unperturbed electron and LO phonon system, H_0, and $H_1 = H_{ep}$ is the electron–phonon interaction Hamiltonian. The energy levels of an electron are shifted over $\Delta E_{N_r,m}$ by the interaction with the LO phonons:

$$E_{N_r,m} = \hbar\tilde{\omega}_c(2N_r + |m| + 1) + \tfrac{1}{2}\hbar\omega_c m + \Delta E_{N_r,m}. \tag{7}$$

Within the second-order perturbation theory the energy shift of the level with the quantum numbers N_r, m is given by

$$\Delta E_{N_r,m} = -\sum_{N_r'=0}^{\infty} \sum_{m'=-\infty}^{\infty} \sum_{\boldsymbol{q}} \frac{|M_{m'm}^{N_r'N_r}(\boldsymbol{q})|^2}{D_{m'm}^{N_r'N_r}}. \tag{8}$$

The matrix element is $M_{m'm}^{N_r'N_r}(\boldsymbol{q}) = \langle N_r', m';\ 1_q|H_{ep}|N_r, m; 0_q\rangle$. The ket $|N_r, m;\ n_q\rangle = |N_r, m\rangle \otimes |n_q\rangle$ describes an unperturbed state of H_0 composed of an electron in the level N_r, m and n LO phonons with the momentum $\hbar\boldsymbol{q}$ and the energy $\hbar\omega_L$ which we denote $(N_r, m; n_q)$. In this paper we only consider weakly polar semiconductors with $\alpha \ll 1$, i.e. we are in the weak-coupling limit and so it is sufficient to consider perturbed states containing not more than one LO phonon. Using the Hamiltonian, equation (5), and the states described above, the matrix element is given by

$$\begin{aligned} M_{m'm}^{N_r'N_r}(\boldsymbol{q}) = {} & \left(\frac{4\pi\alpha r_p(\hbar\omega_L)^2}{V_G}\right)^{1/2} C_{N_r'm'} C_{N_r m} \int_0^\infty \mathrm{d}r \int_0^{2\pi} \mathrm{d}\varphi \exp[\mathrm{i}(m - m')\varphi] \\ & \times r^{|m|+|m'|+1} \exp\left(-\frac{m_e\tilde{\omega}_c}{\hbar}r^2\right) \boldsymbol{F}\left(-N_r', |m'| + 1, \frac{m_e\tilde{\omega}_c}{\hbar}r^2\right) \\ & \times \boldsymbol{F}\left(-N_r, |m| + 1, \frac{m_e\tilde{\omega}_c}{\hbar}r^2\right) \frac{\exp(\mathrm{i}\boldsymbol{q}_\parallel\boldsymbol{x}_\parallel)}{|\boldsymbol{q}|} \end{aligned} \tag{9}$$

with $\boldsymbol{q}_{\|}$ the two-dimensional wave vector in the x–y plane and $q_{\|} = |\boldsymbol{q}_{\|}|$; $r = |\boldsymbol{x}_{\|}|$. The energy denominator in (8) is given by

$$D_{m'm}^{N_r'N_r} = \hbar\omega_L + \hbar\tilde{\omega}_c[2(N_r' - N_r) + |m'| - |m|] + \tfrac{1}{2}\hbar\omega_c(m' - m) - \Delta_{N_r m} \tag{10}$$

where the value $\Delta_{N_r m}$ depends on the type of perturbation theory used [4]: (i) $\Delta_{N_r m} = 0$ leads to the Rayleigh–Schrödinger perturbation theory (RSPT); (ii) $\Delta_{N_r m} = \Delta E_{N_r m}$ results in the Wigner–Brillouin perturbation theory (WBPT); and (iii) $\Delta_{N_r m} = \Delta E_{N_r m} - \Delta E_{00}^{\mathrm{RSPT}}$ gives an improved Wigner–Brillouin perturbation theory (IWBPT), with $\Delta E_{00}^{\mathrm{RSPT}}$ the weak-coupling electron–phonon correction to the electron ground-state energy calculated within RSPT. For the ground state $\Delta E_{00}^{\mathrm{IWBPT}} = \Delta E_{00}^{\mathrm{RSPT}}$ is valid. It is well known [4] that the RSPT desribes the ground-state correction for $\omega_c \to 0$ quite well, but it fails for the excited states, since it is possible that the denominator vanishes for a certain ω_c. This becomes possible if the energy level $(N_r, m; 0_q)$ of the state $|N_r, m; 0_q\rangle$ crosses the energy level $(0, 0; 1_q)$ of the state $|0, 0; 1_q\rangle$ at $\tilde{\omega}_c + m\omega_c/(4N_r + 2|m|) = \omega_L/(2N_r + |m|)$, but the occurrence of a level crossing depends strongly on the relation between the LO phonon frequency ω_L and the confinement frequency Ω. This behaviour is very similar to the case of a QWW with parabolic confinement [10] but different from the 3D and Q2D systems where level crossing always occurs at $N\omega_c = \omega_L$ (N is the number of the Landau level). If resonance occurs, the electron–phonon interaction leads to a splitting of the degenerate levels and a pinning to the energy $\hbar\omega_L + \hbar\tilde{\omega}_c + \Delta E_{00}^{\mathrm{RSPT}}$. The higher-energy branch is not calculated in this paper, because of the condition given by equation (12) below. Only the IWBPT gives the correct pinning behaviour in the weak-coupling limit.

In the experiments the optical transitions $E_{00} \to E_{0\pm1}$ can be used to determine the polaron cyclotron mass $m_c^* = eB/\omega_c^*$. In the following we restrict ourself to the discussion of these three levels. The level crossing between the energy of the state $|0, +1; 0_q\rangle$ and that of $|0, 0; 1_q\rangle$ occurs under the condition $\omega_L > \Omega$ at $\omega_c = (\omega_L^2 - \Omega^2)/\omega_L$ and that between the energy of the state $|0, -1; 0_q\rangle$ and that of $|0, 0; 1_q\rangle$ under the condition $\omega_L < \Omega$ at $\omega_c = (\Omega^2 - \omega_L^2)/\omega_L$. For $\omega_L < \Omega$ in the first case and $\omega_L > \Omega$ in the second there is no level crossing. Therefore, a level crossing occurs either for the transition $E_{00} \to E_{0,+1}$ or for the transition $E_{00} \to E_{0,-1}$.

For the calculation of the polaron cyclotron mass it is necessary to calculate the energy shifts ΔE_{00} and $\Delta E_{0,\pm1}$. Starting with equation (9) for the matrix element $M_{m'0}^{N_r'0}(\boldsymbol{q})$, we obtain

$$|M_{m'0}^{N_r'0}(\boldsymbol{q})|^2 = [(4\pi\alpha r_p(\hbar\omega_L)^2/V_G)/(N_r' + |m'|)!N_r'!] \times (\hbar q_{\|}^2/4m_e\tilde{\omega}_c)^{|m'|+2N_r'}\exp(-\hbar q_{\|}^2/2m_e\tilde{\omega}_c)/|\boldsymbol{q}|^2. \tag{11}$$

Using this result in equation (8) for the energy shift ΔE_{00} it is possible to calculate exactly the sums over the quantum numbers N_r' and m'. We convert the denominator of equation (8) by the integral

$$\frac{1}{D_{m'm}^{N_r'N_r}} = \int_0^\infty \mathrm{d}t\, \exp(-D_{m'm}^{N_r'N_r} t) \tag{12}$$

where $D_{m'm}^{N_r'N_r} > 0$ must be fulfilled. This means that the results are limited to the study of the Landau levels below the level $(0, 0; 1_q)$. Replacing the sum over $\boldsymbol{q}$ by an integral according to

$$\sum_{\boldsymbol{q}} \longrightarrow \frac{V_G}{(2\pi)^3}\int \mathrm{d}^3q = \frac{V_G}{(2\pi)^3}\int_0^\infty \mathrm{d}q_{\|}\, q_{\|}\int_0^{2\pi}\mathrm{d}\varphi\int_{-\infty}^{\infty}\mathrm{d}q_z$$

we obtain

$$\Delta E_{00} = -\alpha r_{\rm p}(\hbar\omega_{\rm L})^2 \int_0^\infty {\rm d}t\ \exp[-(\hbar\omega_{\rm L} - \Delta_{00})t] \sum_{N_{\rm r}'=0}^{\infty} \sum_{m'=-\infty}^{\infty} \frac{1}{(N_{\rm r}' + |m'|)!N_{\rm r}'!}$$
$$\times \int_0^\infty {\rm d}q_\parallel \left(\frac{\hbar q_\parallel^2}{4m_{\rm e}\tilde{\omega}_{\rm c}}\right)^{2N_{\rm r}'+|m'|} \exp\left(-\frac{\hbar q_\parallel^2}{2m_{\rm e}\tilde{\omega}_{\rm c}}\right)$$
$$\times \exp\{-[(2N_{\rm r}' + |m'|)\hbar\tilde{\omega}_{\rm c} + \tfrac{1}{2}\hbar\omega_{\rm c} m']t\}. \tag{13}$$

To perform the sum over the quantum numbers $N_{\rm r}'$ and m' we introduce the new quantum numbers p and s, defined by $p + s = |m'| + 2N_{\rm r}'$ and $p - s = m'$. Consequently, we have for $m' \geqslant 0$, $p = N_{\rm r}' + m'$ and $s = N_{\rm r}'$ and for $m' < 0$, $p = N_{\rm r}'$ and $s = N_{\rm r}' - m'$. If $N_{\rm r}' \to \infty$ and $-\infty < m' < \infty$, the new quantum numbers s and p go independently from 0 to ∞. The result is that all terms are factorized and we finally obtain

$$\Delta E_{00} = -\frac{\pi^{1/2}\alpha r_{\rm p}(\hbar\omega_{\rm L})^2}{2}$$
$$\times \int_0^\infty {\rm d}t\ \exp[-(\hbar\omega_{\rm L} - \Delta_{00})t] \bigg/ \left(\frac{\hbar}{2m_{\rm e}\tilde{\omega}_{\rm c}}[1 - \exp(-\hbar\tilde{\omega}_{\rm c}t)\cosh(\tfrac{1}{2}\hbar\omega_{\rm c}t)]\right)^{1/2}. \tag{14}$$

Thus the interesting energy correction is given by a 1D integral.

Now we consider the energy shift $\Delta E_{0,\pm1}$. We again start with equation (9) for the matrix element $M_{m'\pm1}^{N_{\rm r}'0}(q)$. This matrix element is simply calculated and reads

$$|M_{m'\pm1}^{N_{\rm r}'0}(q)|^2 = [(4\pi\alpha r_{\rm p}(\hbar\omega_{\rm L})^2/V_{\rm G})/(N_{\rm r}' + |m'|)!N_{\rm r}'!]$$
$$\times (\hbar q_\parallel^2/4m_{\rm e}\tilde{\omega}_{\rm c})^{|m'|+2N_{\rm r}'-1}[\tfrac{1}{2}|m'| + N_{\rm r}' - \hbar q_\parallel^2/4m_{\rm e}\tilde{\omega}_{\rm c}]^2 \exp(-\hbar q_\parallel^2/2m_{\rm e}\tilde{\omega}_{\rm c})/|q|^2. \tag{15}$$

Using this expression in equation (8) where we replace the denominator again by an integral, equation (12), and introducing again the new quantum numbers p and s, we can exactly carry out both sums over p and s. The resulting expression for the energy shift reads

$$\Delta E_{0\pm1} = -\frac{\pi^{1/2}\alpha r_{\rm p}(\hbar\omega_{\rm L})^2}{8} \int_0^\infty {\rm d}t\ \exp[-(\hbar\omega_{\rm L} - \hbar\tilde{\omega}_{\rm c} \mp \tfrac{1}{2}\hbar\omega_{\rm c} - \Delta_{0\pm1})t]$$
$$\times [1 + \exp(-\hbar\tilde{\omega}_{\rm c}t)\cosh(\tfrac{1}{2}\hbar\omega_{\rm c}t)] \bigg/ \left(\frac{\hbar}{2m_{\rm e}\tilde{\omega}_{\rm c}}[1 - \exp(-\hbar\tilde{\omega}_{\rm c}t)\right.$$
$$\left. \times \cosh(\tfrac{1}{2}\hbar\omega_{\rm c}t)]\right)^{1/2} \tag{16}$$

and we have again only a remaining 1D integral. The expressions for ΔE_{00}, equation (14), and $\Delta E_{0\pm1}$, equation (16), are valid for all magnetic fields because in our calculation no simplifying approximations are made.

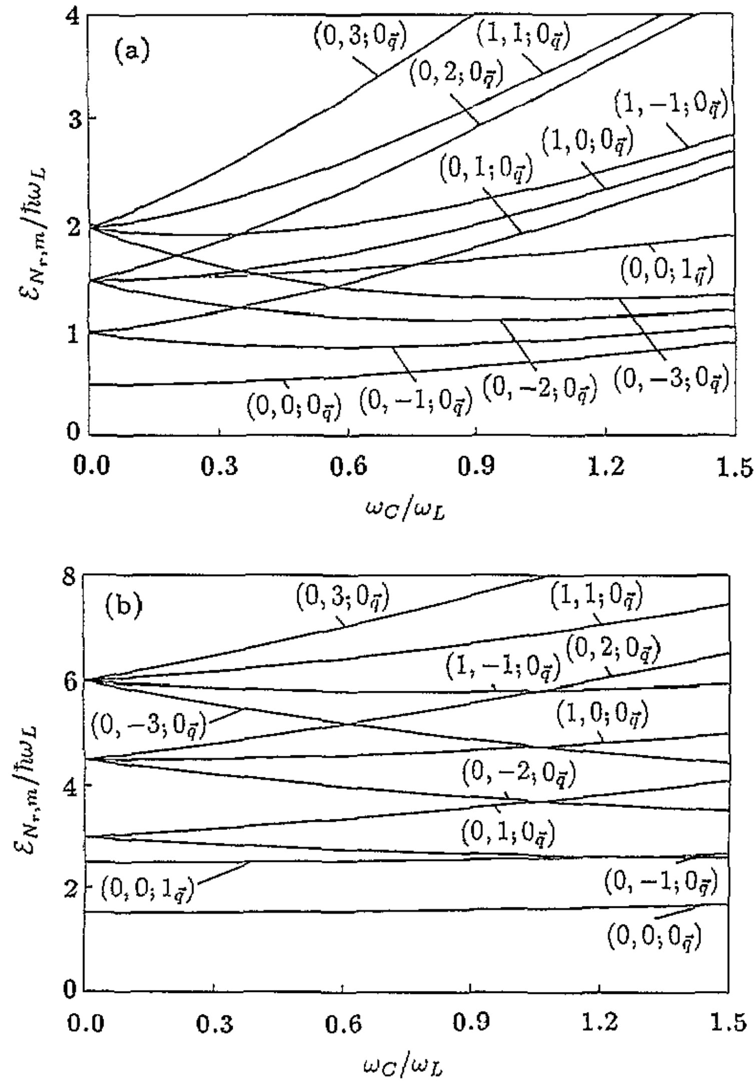

Figure 1. The first unperturbed Landau levels $(N_r, m; n_q)$ as a function of the magnetic field in a GaAs–$Ga_{0.75}Al_{0.25}As$ QD (bold curves). The fine curve corresponds to the unperturbed level $(0, 0; 1_q)$. The Landau levels are plotted in figure 1(a) for $\Omega = 0.5\omega_L$ and in figure 1(b) for $\Omega = 1.5\omega_L$.

3. Numerical results for the polaron correction to the Landau levels

For numerical calculation we have used a nanostructured GaAs–$Ga_{1-x}Al_xAs$ heterostructure (GaAs: $\alpha = 0.07$, $r_p = 3.987$ nm, $\hbar\omega_L = 36.17$ meV, $m_e = 0.06624 m_0$) in which the electrons are confined within GaAs. Because in the experimentally realized structures typically $\hbar\Omega \leqslant 4$ meV for GaAs is valid we have the resonance for the transition $E_{00} \to E_{0+1}$ at $\omega_c = (\omega_L^2 - \Omega^2)/\omega_L$.

In figure 1 the unperturbed Landau levels for one electron in the QD are plotted as a function of the magnetic field for two different confinement potentials: (a) $\Omega = 0.5\omega_L$ and (b) $\Omega = 1.5\omega_L$. For $B = 0$ the levels are $(N + 1)$-fold degenerate at the 2D harmonic oscillator energy levels $\mathcal{E}_N = \hbar\Omega(N+1)$ with $N = 2N_r + |m|$. In the opposite limit, $B \to \infty$, the Landau levels of the electron in a QD approach those of the 2D system: $\mathcal{E}_N^{2D} = \hbar\omega_c(N+\frac{1}{2})$ with $N = N_r + \frac{1}{2}(|m|+m)$. Consequently, for $B \to \infty$ each Landau level is degenerate (without spin degeneracy) according to the degeneracy factor $N_L = (eB/h)A$ with $A = L_x L_y$ the area of the x–y plane. In the case of $\omega_L > \Omega$, figure 1(a), the level crossing occurs between the states $|0, 0; 1_q\rangle$ and $|0, +1; 0_q\rangle$ under consideration at $\omega_c = (\omega_L^2 - \Omega^2)/\omega_L$. But this figure also shows that higher levels cross the level $(0, 0; 1_q)$. This is possible because the levels with negative m and $N_r = 0$ have a negative slope for small magnetic fields and approach the lowest Landau level of 2D electrons for large magnetic fields. In figure 1(a) such a level crossing occurs between the states $|0, 0; 1_q\rangle$ and $|0, m; 0_q\rangle$ with $m < -2$. In figure 1(b) the situation $\omega_L < \Omega$ is plotted. In this case the level crossing occurs between the states $|0, 0; 1_q\rangle$ and $|0, m; 0_q\rangle$ with $m < 0$. It is evident that the crossing occurs for larger $|m|$ at larger magnetic fields. Hence, we have two different types of level crossing: with increasing magnetic field, the state $|0, 0; 1_q\rangle$ is crossed from a level (i) from the lower energy side and (ii) from the higher energy side. From the material parameters of GaAs it is obvious that the situation plotted in figure 1(a) can only be experimentally realized up to now. All following calculations and conclusions are valid for this case $\Omega < \omega_L$.

The calculated Landau levels for one electron in different QDs including polaron effects are plotted in figure 2. The fine full curves show the unperturbed levels $|N_r, m; 0_q\rangle$, the fine broken curves the unperturbed level $|0, 0; 1_q\rangle$ and the bold curves are the corresponding perturbed levels. The perturbed levels are obtained from equations (14) and (16), respectively. From figure 2 it is apparent that (i) the perturbed levels, the magnetopolaron levels, are shifted to lower energies $\simeq \Delta E_{N_r m}(\omega_c = 0)$ independent of the magnetic field and (ii) with increasing magnetic field the state $|0, +1; 0_q\rangle$ mixes strongly with $|0, 0; 1_q\rangle$, becoming resonant near the unperturbed level crossing at $\omega_c = (\omega_L^2 - \Omega^2)/\omega_L$. The Landau levels are repelled from the level $(0, 0; 1_q)$ and pinned to the energy $\hbar\omega_L + \hbar\tilde{\omega}_c + \Delta E_{00}^{\mathrm{RSPT}}$ in the following manner. The levels for which, at $B = 0$, $\mathcal{E}_{N_r m} = \hbar\Omega(2N_r + |m| + 1) < \hbar(\omega_L + \Omega)$ is valid, are pinned to this level from the lower-energy side, but the levels crossing the level $(0, 0; 1_q)$ at a certain magnetic field and for which, at $B = 0$, $\mathcal{E}_{N_r m} = \hbar\Omega(2N_r + |m| + 1) > \hbar(\omega_L + \Omega)$ is valid, are pinned from the higher-energy side to the level $(0, 0; 1_q)$. According to the condition for the validity of equation (12) we are limited to the study of the polaron correction to the states below the energy of the state $|0, 0; 1_q\rangle$. Figure 2 shows that the polaron correction to the Landau levels at $B = 0$ decreases with increasing quantum number $(N_r, |m|)$. This behaviour is different from the well known 3D and 2D magnetopolaron [6] for which the polaron corrections at $B = 0$ are independent of the quantum number N. Comparing figures 2(a) and 2(b) one can see that with increasing Ω of the confinement potential the mixing of the levels $|0, 0; 1_q\rangle$ and $|0, +1; 0_q\rangle$ becomes stronger for smaller magnetic fields. For the chosen examples the crossing of the unperturbed levels occurs at the magnetic field $B = B_c$ for the GaAs–$Ga_{1-x}Al_xAs$ QD with $B_c = 20.4$ T for $\hbar\Omega = 4$ meV and $B_c = 18.4$ T for $\hbar\Omega = 12$ meV.

In cyclotron resonance experiments the transition of electrons between the Landau levels is observed. Hence, the transition is detected between the plotted magnetopolaron levels (perturbed or renormalized levels) of figure 2. The energy difference between the two successive Landau levels $E_{0\pm1} - E_{00} = \hbar(\tilde{\omega}_c \pm \frac{1}{2}\omega_c) + \Delta E_{0\pm1} - \Delta E_{00}$ is plotted in figure 3 as a function of the magnetic field for different GaAs–$Ga_{1-x}Al_xAs$ QDs. The bold curves represent the renormalized energy difference and the fine curves those of the unperturbed levels. Because $\Delta E_{0+1} - \Delta E_{00} > 0$ at $B = 0$, the energy difference is larger between the renormalized levels than between the unperturbed levels. The energy difference $E_{0+1} - E_{00}$

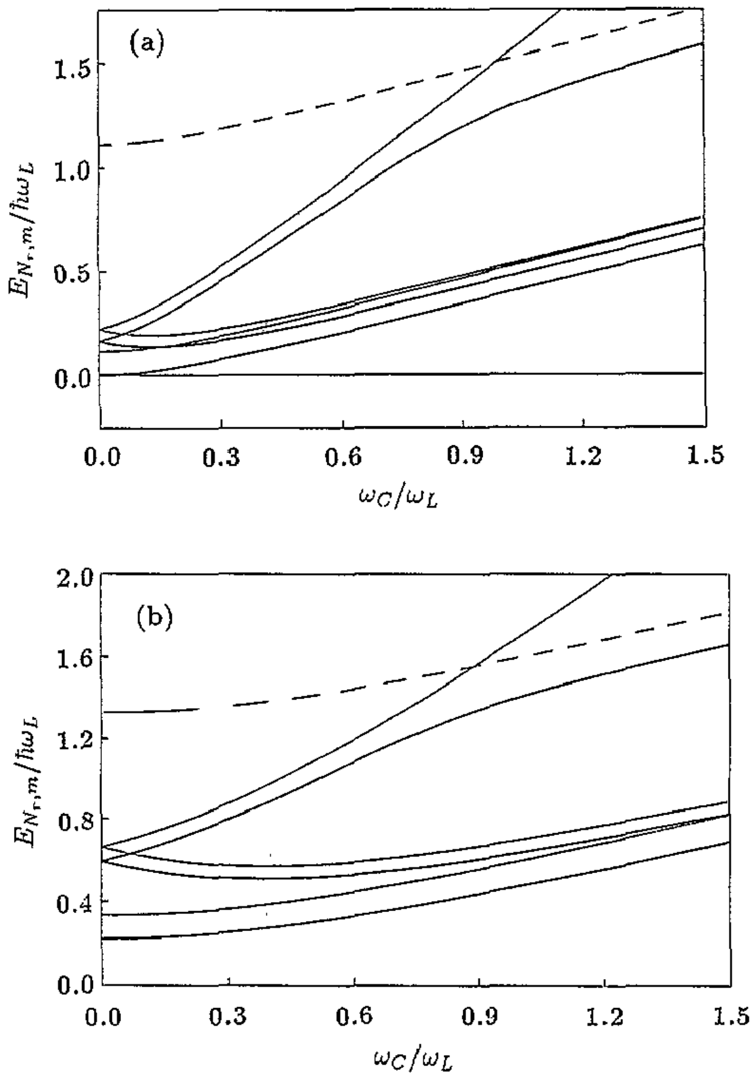

Figure 2. The first magnetopolaron levels E_{00}, $E_{0\pm}$ (bold curves) as a function of the magnetic field in a GaAs–$Ga_{0.75}Al_{0.25}$As QD for $\hbar\Omega = 4$ meV (a) and $\hbar\Omega = 12$ meV (b). The corresponding unperturbed Landau levels are plotted by fine full $(0, 0; 0_q)$, $(0, \pm 1; 0_q)$ and broken $(0, 0; 1_q)$ curves.

becomes equal to the LO phonon energy $\hbar\omega_L$ in the limit $B \to \infty$, whereas $E_{0-1} - E_{00}$ becomes zero in this limit. For the GaAs QDs [15] there are two possibilities for measuring the electron–phonon-interaction-induced resonance effects by new experiments:

(i) For the configuration of Meurer *et al* [15] with a subband separation of only $\hbar\Omega = 1.6$ meV it is necessary to use higher magnetic fields of $B_c \approx 21$ T.

(ii) If one could produce QDs with larger subband separations of for instance $\hbar\Omega = \frac{1}{2}\hbar\omega_L \approx 18$ meV one could reduce the necessary critical magnetic field to $B_c \approx 17$ T to measure the resonance effect.

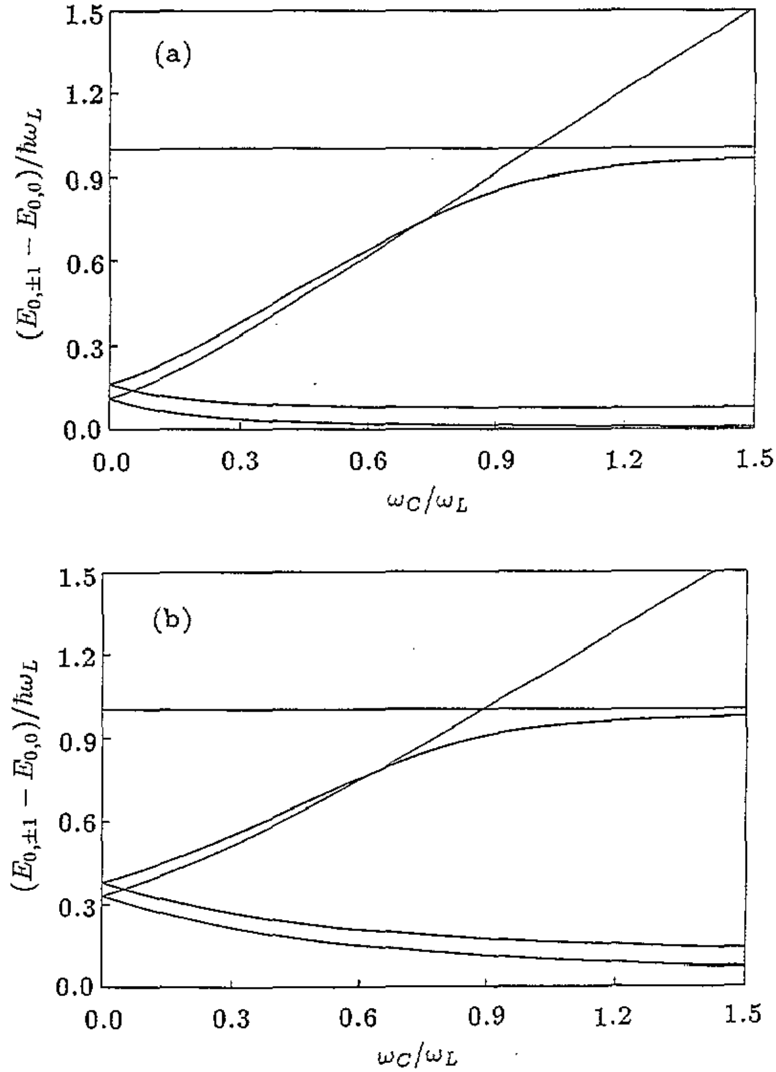

Figure 3. Energy differences $E_{0+1} - E_{00}$ and $E_{0-1} - E_{00}$ between the Landau levels for the perturbed states (magnetopolaron, bold curves) and the unperturbed states (electron, fine curves) for a GaAs–$Ga_{0.75}Al_{0.25}As$ QD with $\hbar\Omega = 4$ meV (a) and $\hbar\Omega = 12$ meV (b).

4. Polaron cyclotron mass and analytic results for weak magnetic field

4.1. Polaron cyclotron mass in quantum dots

In the experiments the optical transition $E_{00} \to E_{0\pm1}$ can be used to determine the polaron cyclotron mass m_c^*. If one uses the transition $E_{00} \to E_{0+1}$ the cyclotron mass of a resonant magnetopolaron is obtained. The corresponding mass can be calculated by

$$m_c^* = e\hbar B/[E_{0+1} - E_{00} - \hbar^2\Omega^2/(E_{0+1} - E_{00})]. \tag{17}$$

If on the other hand only the transition $E_{00} - E_{0-1}$ is detected, the resulting effective mass of a non-resonant magnetopolaron is obtained. This mass is given by

$$m_c^* = e\hbar B/[\hbar^2\Omega^2/(E_{0-1} - E_{00}) - (E_{0-1} - E_{00})]. \tag{18}$$

If both transitions are measured, it is possible to define the polaron cyclotron mass according to

$$m_c^* = e\hbar B/(E_{0+1} - E_{0-1}). \tag{19}$$

This definition also provides the resonant magnetopolaron. It is obvious that the polaron cyclotron mass depends on the used optical transition. If one compares the definitions of the polaron cyclotron mass of the magnetopolaron, (17)–(19), one finds the following physical differences. For this discussion we need first the limits of ΔE_{00} and $\Delta E_{0\pm1}$ for weak magnetic fields and weak lateral confinement. If $\omega_c/\omega_L \ll 1$ is valid, non-degenerate perturbation theory, i.e. RSPT, has to be used and, consequently, one has $\Delta_{N,M} = 0$. If one expands ΔE_{00} and $\Delta E_{0\pm1}$ in a power series according to $\xi = \omega_c/\omega_L$, we obtain for the electron–phonon correction (14) to the Landau level E_{00}

$$\begin{aligned}\Delta E_{00} = -\tfrac{1}{2}\alpha\hbar\omega_L\pi([\Gamma(1/\eta)\eta^{-1/2}/\Gamma(1/\eta+\tfrac{1}{2})]\xi^0 + [\Gamma(1/\eta)\eta^{-5/2}/16\Gamma(1/\eta+\tfrac{1}{2})]\\ \times [\![1 - 2\eta^{-1}\{[\Psi(1/\eta+1) - \Psi(1/\eta+\tfrac{1}{2})] + [\Psi(1/\eta+1) - \Psi(1/\eta+\tfrac{1}{2})]^2\\ + [\Psi'(1/\eta+1) - \Psi'(1/\eta+1/2)]\}]\!]\xi^2 + \mathcal{O}(\xi^4)).\end{aligned} \tag{20}$$

Herein $\eta = \Omega/\omega_L$, $\Gamma(x)$ is the gamma function and $\Psi(x)$ is the psi function with $\Psi'(x)$ its derivative. Note that odd powers of ξ are absent for the ground-state renormalization in contrast to the 3D and 2D magnetopolaron. For a weak confinement energy, $\Omega \ll \omega_L$, equation (20) can be further expanded in a power series of η:

$$\Delta E_{00} = -\tfrac{1}{2}\alpha\hbar\omega_L\pi\{[1 + \tfrac{1}{8}\eta + \tfrac{1}{128}\eta^2 + \mathcal{O}(\eta^3)]\xi^0 + (3/64\eta)[1 - \tfrac{5}{24}\eta + \mathcal{O}(\eta^2)]\xi^2 + \mathcal{O}(\xi^4)\}. \tag{21}$$

We note that this expression for the ground-state energy contains the result $\Delta E_{00} = -\frac{1}{2}\alpha\hbar\omega_L\pi$ for $\omega_c \to 0$ and $\Omega \to 0$, which is the correct energy correction of a strict 2D polaron within second-order RSPT.

The corrections $\Delta E_{0\pm1}$, equation (16), due to electron–phonon interaction to the energy levels $\mathcal{E}_{0\pm1}$ for a weak magnetic field are given by

$$\begin{aligned}\Delta E_{0\pm1} = -\tfrac{1}{4}\alpha\hbar\omega_L\pi([\Gamma(1/\eta)\eta^{-1/2}/\Gamma(1/\eta+\tfrac{1}{2})][(1-\tfrac{3}{4}\eta)/(1-\eta)]\xi^0\\ \pm\{[\Gamma(1/\eta)\eta^{-3/2}/4\Gamma(1/\eta+\tfrac{1}{2})]/2(1-\eta)^2\}\{(4-3\eta)(1-\eta)[\Psi(1/\eta+\tfrac{1}{2})\\ - \Psi(1/\eta)] + \eta^2\}\xi^1 + \mathcal{O}(\xi^2)).\end{aligned} \tag{22}$$

Note that the electron–phonon corrections $\Delta E_{0\pm1}$ for the energy levels $\mathcal{E}_{0\pm1}$ are only different from one another in the sign of the odd powers of ξ. In the case of a weak confinement potential we obtain from equation (22)

$$\Delta E_{0\pm1} = -\tfrac{1}{4}\alpha\hbar\omega_L\pi\{[1 + \tfrac{3}{8}\eta + \tfrac{37}{128}\eta^2 + \mathcal{O}(\eta^3)]\xi^0 \pm \tfrac{1}{4}[1 + \tfrac{9}{8}\eta + \tfrac{185}{128}\eta^2 + \mathcal{O}(\eta^3)]\xi^1 + \mathcal{O}(\xi^2)\}. \tag{23}$$

Following the electron–phonon contribution to the splitting between two successive Landau levels reads:

$$\begin{aligned}\Delta E_{0\pm1} - \Delta E_{00} = \tfrac{1}{4}\alpha\hbar\omega_L\pi\{[1 - \tfrac{1}{8}\eta - \tfrac{35}{128}\eta^2 + \mathcal{O}(\eta^3)]\xi^0\\ \mp \tfrac{1}{4}[1 + \tfrac{9}{8}\eta + \tfrac{185}{128}\eta^2 + \mathcal{O}(\eta^3)]\xi^1 + \mathcal{O}(\xi^2)\}\end{aligned} \tag{24}$$

$$\Delta E_{0+1} - \Delta E_{0-1} = -\tfrac{1}{8}\alpha\hbar\omega_L\pi\{[1 + \tfrac{9}{8}\eta + \tfrac{185}{128}\eta^2 + \mathcal{O}(\eta^3)]\xi^1 + \mathcal{O}(\xi^3)\}. \tag{25}$$

These results allow us to discuss the physical meaning of the different polaron cyclotron masses. The masses defined in equations (17)–(19) result for vanishing electron–phonon interaction $\alpha \to 0$ in the ordinary electron conduction band-edge mass: $m_c^* \to m_e$. For vanishing magnetic field the masses m_c^* defined by equations (17) and (18) vanish. This is true because the denominator of the right-hand side of equations (17) and (18) is a constant for $B \to 0$. In contrast to this unusual behaviour of the polaron cyclotron mass, defined by equations (17) and (18), that defined by equation (19) reaches, for $B \to 0$

$$m_c^*/m_e = (1 - \tfrac{1}{8}\alpha\pi [\![[\Gamma(1/\eta)\eta^{-3/2}/\Gamma(1/\eta + \tfrac{1}{2})2(1-\eta)^2] \times \{(4-3\eta)(1-\eta)[\Psi(1/\eta + \tfrac{1}{2}) - \Psi(1/\eta)] + \eta^2\}\xi^0 + \mathcal{O}(\xi^2)]\!])^{-1} \tag{26}$$

which contains for weak confinement energy, $\Omega \ll \omega_L$, the result

$$m_c^*/m_e = 1/[\![1 - \tfrac{1}{8}\alpha\pi\{[1 + \tfrac{9}{8}\eta + \tfrac{185}{128}\eta^2 + \mathcal{O}(\eta^3)]\xi^0 + \mathcal{O}(\xi^2)\}]\!]. \tag{27}$$

Hence, we have the correct effective-mass correction of a strict 2D polaron $(m^*/m_e) = 1/(1 - \frac{1}{8}\alpha\pi)$. For this reason we use the polaron cyclotron mass, defined in equation (19), because this definition provides the correct limiting cases. We note that for zero magnetic field and finite confinement ($\Omega > 0$) it is impossible to define a polaron mass, which has a kinetic nature, because of the entirely quantized spectrum of the polaron, but for $B > 0$, classically, the electrons move on skipping orbits along the edge of the QD due to the lateral confinement and the Lorentz force.

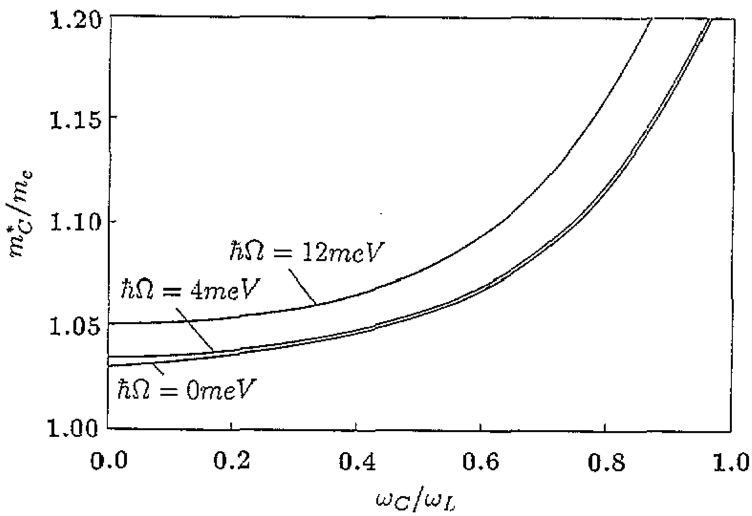

Figure 4. Polaron cyclotron mass of the Q0D magnetopolaron versus magnetic field for a GaAs-$Ga_{0.75}Al_{0.25}As$ QD for different confinement energies $\hbar\Omega = 12$ meV, $\hbar\Omega = 4$ meV and $\hbar\Omega = 0$ meV corresponding to the polaron cyclotron mass of a 2D magnetopolaron.

This Q0D polaron cyclotron mass, defined in equation (19), is shown in figure 4. The increase of the mass with increasing magnetic field is the polaron-induced non-parabolicity in the absence of the conduction-band non-parabolicity. The strong enhancement of the polaron cyclotron mass around $\omega_c = (\omega_L^2 - \Omega^2)/\omega_L$ is a consequence of the pinning of the

Landau level E_{0+1} to the energy $\hbar\omega_L + \hbar\breve{\omega}_c + \Delta E_{00}^{RSPT}$. If one compares the results for RSPT, WBPT and IWBPT, figure 5, we can conclude that for Q0D systems the same differences between the different types of perturbation theory are valid as for 3D, 2D and Q1D systems [4, 10, 16]: RSPT overestimates the contribution of the polaron effects to the polaron mass near the resonance and WBPT underestimates the polaron effects whereas IWBPT is a good improvement to WBPT.

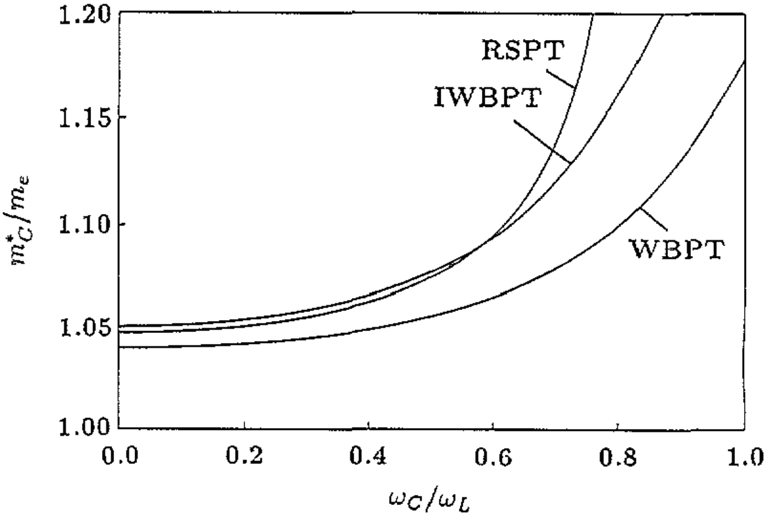

Figure 5. Polaron cyclotron mass of the Q0D magnetopolaron versus magnetic field for a GaAs-$Ga_{0.75}Al_{0.25}As$ QD for different types of perturbation theory, RSPT, WBPT and IWBPT, for $\hbar\Omega = 12$ meV.

4.2. *Comparison of magnetopolarons from 3D to Q0D*

The energy levels of an electron in a magnetic field interacting with 3D bulk LO phonons are

$$\begin{aligned} E_N(k_z) &= \hbar\omega_c(N+\tfrac{1}{2}) + \hbar^2 k_z^2/2m_e + \Delta E_N(k_z) && \text{in 3D} \\ E_N &= \hbar\omega_c(N+\tfrac{1}{2}) + \Delta E_N && \text{in 2D} \\ E_N(k_x) &= \hbar\breve{\omega}_c(N+\tfrac{1}{2}) + \hbar^2 k_x^2/2\breve{m} + \Delta E_N(k_x) && \text{in Q1D} \\ E_{N_r,m} &= \hbar\breve{\omega}_c(2N_r + |m| + 1) + \tfrac{1}{2}\hbar\omega_c m + \Delta E_{N_r,m} && \text{in Q0D} \end{aligned} \tag{28}$$

where $\breve{\omega}_c = (\omega_c^2 + \Omega^2)^{1/2}$ is the hybrid frequency and $\breve{m} = m_e(\breve{\omega}_c/\Omega)^2$. In the Q1D case we have assumed a parabolic confinement potential in the lateral direction: $V(y) = \frac{1}{2}m_e\Omega^2 y^2$ and a strict confinement in the z direction. In the 3D and in the 2D case, each Landau level is degenerated according to the degeneracy factor N_L. From equation (28) it follows that in 3D and Q1D semiconductor systems there exists a phonon continuum which has a threshold energy of $E_{th} = \hbar\omega_L + \frac{1}{2}\hbar\omega_c + \Delta E_0$ and $E_{th} = \hbar\omega_L + \frac{1}{2}\hbar\breve{\omega}_c + \Delta E_0$, respectively, but in the 2D and Q0D case there is no phonon continuum in the spectrum. The polaron cyclotron resonance frequency $\hbar\omega_c^* = E_N - E_{N-1}$ in 3D and 2D, and the polaron hybrid frequency $\hbar\breve{\omega}_c^* = E_N - E_{N-1}$ in Q1D, define a polaron cyclotron mass according to

$$m_c^* = e\hbar B/(E_N - E_{N-1}) \qquad \text{in 3D and 2D} \tag{29a}$$

and [10]

$$m_c^* = e\hbar B/[(E_N - E_{N-1})^2 - (\hbar\Omega)^2]^{1/2} \qquad \text{in Q1D.} \tag{29b}$$

The corresponding expressions for a Q0D magnetopolaron are given in equations (17)–(19). In the weak-magnetic-field limit $\omega_c/\omega_L \ll 1$ the electron–phonon correction in second-order RSPT to the Landau level $E_N(0)$ in 3D is given by [6]

$$\Delta E_N(0) = -\alpha\hbar\omega_L\{1 + \tfrac{1}{12}(2N+1)\xi + \tfrac{1}{240}(18N^2 + 18N - 1)\xi^2 + [(90N^3 + 135N^2 - 37N - 6)/2016]\xi^3 + \mathcal{O}(\xi^4)\}. \tag{30}$$

Consequently, the electron–phonon contribution to the splitting between two successive Landau levels reads

$$\Delta E_N(0) - \Delta E_{N-1}(0) = -\tfrac{1}{6}\alpha\hbar\omega_L\xi\{1 + \tfrac{9}{10}N\xi + \tfrac{1}{168}[135(N-1)(N+1) + 94]\xi^2 + \mathcal{O}(\xi^3)\}. \tag{31}$$

If we use the transition $E_0 \to E_1$ to obtain the polaron cyclotron mass, the mass is given for small magnetic fields by

$$m_c^*/m_e = 1/\{1 - \tfrac{1}{6}\alpha(1 + \tfrac{9}{10}\xi + \tfrac{47}{84}\xi^2 + \mathcal{O}(\xi^3))\}. \tag{32}$$

This expression contains for $B \to 0$ the well known 3D polaron mass.

The analogous results for the strict 2D magnetopolarons read [6]

$$\Delta E_N = -\tfrac{1}{2}\alpha\hbar\omega_L\pi\{1 + \tfrac{1}{8}(2N+1)\xi + \tfrac{1}{128}[18N(N+1) + 1]\xi^2 + [5(2N+1)(10N^2 + 10N - 1)/1024]\xi^3 + \mathcal{O}(\xi^4)\} \tag{33}$$

$$\Delta E_N - \Delta E_{N-1} = -\tfrac{1}{8}\alpha\hbar\omega_L\pi\xi\{1 + \tfrac{9}{8}N\xi + \tfrac{5}{128}[30(N-1)^2 + 60(N-1) + 29]\xi^2 + \mathcal{O}(\xi^3)\} \tag{34}$$

$$m_c^*/m_e = 1/\{1 - \tfrac{1}{8}\alpha\pi[1 + \tfrac{9}{8}\xi + \tfrac{145}{128}\xi^2 + \mathcal{O}(\xi^3)]\} \tag{35}$$

which yield for $B \to 0$ the well known result for the 2D polaron mass.

In the weak-magnetic-field limit the electron–phonon correction in second-order RSPT to the first two Landau levels $E_0(0)$ and $E_1(0)$ in a Q1D QWW is given by [10]

$$\Delta E_0(0) = -\alpha\hbar\omega_L\frac{1}{\sqrt{\pi}\sqrt{\eta}}\Bigg[\int_0^\infty dt\,\exp\left(-\frac{t}{\eta}\right)\frac{1}{\sqrt{t}}K(t_1)\xi^0 + \int_0^\infty dt\,\frac{\exp(-t/\eta)}{4\eta^2\sqrt{t}}\Bigg(\frac{2 - 5t + 2t^2 - (4-6t)\exp(-t) + (2-t)\exp(-2t)}{t[\exp(-t) - 1 + t]} \times K(t_1) + \frac{2 - 2t - (4 - 3t + t^2)\exp(-t) + (2-t)\exp(-2t)}{t^{3/2}[\exp(-t) - 1 + t]^{1/2}}K'(t_1)\Bigg)\xi^2 + \mathcal{O}(\xi^4)\Bigg] \tag{36}$$

and

$$\Delta E_1(0) - \Delta E_0(0) = -\alpha\hbar\omega_{\rm L}\frac{2}{\sqrt{\pi}\sqrt{\eta}}\bigg[\int_0^\infty {\rm d}t\, \exp\left(-\frac{t}{\eta}\right)\frac{\sinh^2\frac{1}{2}t}{t} \times \frac{1}{\sqrt{\exp(-t)-1+t}}K'(t_1)\xi^0 + \mathcal{O}(\xi^2)\bigg] \tag{37}$$

with

$$t_1 = \sqrt{\exp(-t)-1+t}/\sqrt{t}.$$

Herein $K(x)$ is the complete elliptic integral of the first kind and $K'(x)$ its first derivative. In the case of weak confinement potential, we obtain for vanishing magnetic field

$$\Delta E_0(0) = -\tfrac{1}{2}\alpha\hbar\omega_{\rm L}\pi[1 + (1/4\sqrt{2\pi})\eta^{1/2} + \tfrac{9}{256}\eta + (11/1536\sqrt{2\pi})\eta^{3/2} - (933/262\,144)\eta^2 + \mathcal{O}(\eta^{5/2})] \tag{38}$$

and

$$\Delta E_1(0) - \Delta E_0(0) = -\alpha\hbar\omega_{\rm L}\tfrac{1}{16}\sqrt{2\pi}\eta^{1/2}[1 + \tfrac{9}{32}\sqrt{2\pi}\eta^{1/2} + \tfrac{289}{384}\eta + (3675/16\,384)\sqrt{2\pi}\eta^{3/2} + \mathcal{O}(\eta^2)]. \tag{39}$$

The polaron cyclotron mass in the weak-magnetic-field limit is [10]

$$\frac{m_{\rm c}^*}{m_{\rm e}} = \xi\bigg\{-\alpha\frac{2\hbar\omega_{\rm L}}{\sqrt{\pi}\sqrt{\eta}}\int_0^\infty {\rm d}t\,\exp\left(-\frac{t}{\eta}\right)\frac{\sinh^2\frac{1}{2}t}{t[\exp(-t)-1+t]^{1/2}}K'(t_1) \times\left[2\eta - \alpha\frac{2\hbar\omega_{\rm L}}{\sqrt{\pi}\sqrt{\eta}}\int_0^\infty {\rm d}t\,\exp\left(-\frac{t}{\eta}\right)\frac{\sinh^2\frac{1}{2}t}{t[\exp(-t)-1+t]^{1/2}}K'(t_1)\right]\xi^0 + \mathcal{O}(\xi^2)\bigg\}^{-1/2}. \tag{40}$$

If we compare the electron–phonon corrections in second-order RSPT to the Landau levels in the weak-magnetic-field limit for the dimensionalities 3D to Q0D, it is to be seen that the power series on $\xi = \omega_{\rm c}/\omega_{\rm L}$ contain all powers for 3D, equation (30), 2D, equation (33), and for the Landau levels $\mathcal{E}_{0\pm1}$ of Q0D, equation (22), but only even powers of ξ for the Landau level $\mathcal{E}_{00}$ of the Q0D case, equation (20), and for all levels of the Q1D case, equations (36) and (37). Note that the polaron corrections in 3D and 2D are larger for higher Landau levels for finite magnetic fields but equal for vanishing magnetic field. For Q1D the polaron correction to the Landau levels is different for all levels including the case of vanishing magnetic field, equations (37) and (39). This is also true for the Q0D Landau levels $\mathcal{E}_{N_r,m}$ with different N ($N = 2N_r + |m|$). The levels with the same value N, but a different combination of radial and azimuthal quantum numbers, N_r and m, have the same electron–phonon correction for $B \to 0$. The electron–phonon contribution to the energy splitting between two successive Landau levels vanishes for vanishing magnetic field ($\xi \to 0$) in 3D, equation (31), in 2D, equation (34), and in Q0D for $\Delta E_{0+1} - \Delta E_{0-1}$, equation (25), but this difference remains finite in Q0D for $\Delta E_{0\pm1} - \Delta E_{00}$, equation (24), and in Q1D, equation (37). This different

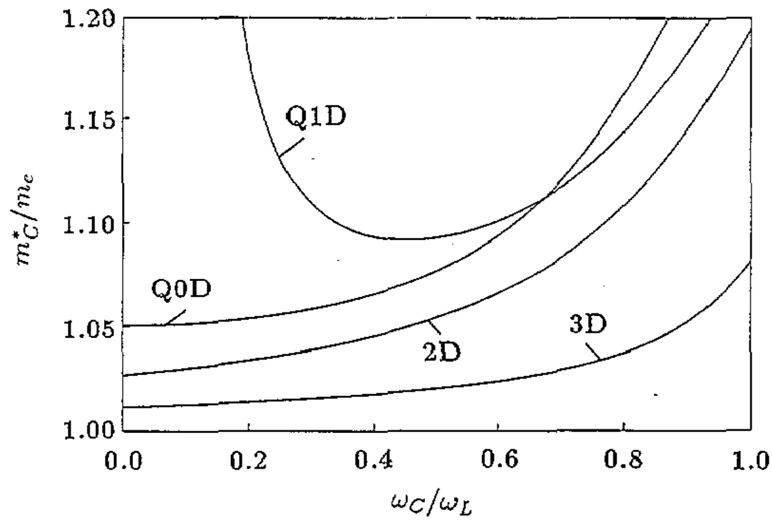

Figure 6. Polaron cyclotron mass of the 3D, 2D, Q1D and Q0D magnetopolaron versus magnetic field for a GaAs-$Ga_{0.75}Al_{0.25}As$ heterostructure. For the Q1D and Q0D magnetopolaron $\hbar\Omega = 12$ meV is used.

behaviour of the renormalization of the energy difference of two successive Landau levels due to electron–phonon coupling results in differencies of the polaron cyclotron mass. It is obvious that in the case $B \to 0$ the polaron cyclotron mass results only in the polaron mass if the electron–phonon contribution to the energy splitting vanishes. Hence, in the cyclotron-resonance experiment it is possible to measure the polaron mass in 3D and 2D and in Q0D only if one uses both transitions, $E_{00} \to E_{0+1}$ and $E_{00} \to E_{0-1}$, but in the Q1D and the Q0D only using one transition, it is not possible to measure the polaron mass.

In figure 6 we plotted the polaron cyclotron mass (calculated within IWBPT) for the 3D, 2D, Q1D and Q0D magnetopolaron versus the magnetic field. This figure shows clearly the enhancement of the mass renormalization due to polaronic effects on reducing the dimensionality. The Q1D polaron cyclotron mass increases with decreasing magnetic field for small magnetic fields resulting from the geometrical confinement. The geometrical confinement also results in a different renormalization of the different Landau levels in the limit $B \to 0$. Hence, there exists a critical cyclotron frequency [10], which is a function of the geometrical confinement frequency Ω, defining a lower bound for the magnetic field to measure the polaron cyclotron mass in the experiment.

5. Conclusions

We have calculated the polaron corrections to the Landau levels and cyclotron masses of Q0D magnetopolarons in QDs. Our results are valid for zero temperature and arbitrary magnetic field strength. It is shown that the Landau levels $\mathcal{E}_{N_r,m}$ with different $N = 2N_r + |m|$ have different polaron corrections for all magnetic fields, but the levels with the same quantum number N have the same energy correction for $B \to 0$. Hence, the electron–phonon contribution to the energy splitting between two successive Landau levels differs for vanishing magnetic fields in dependence on the combined quantum number N of both levels. According to the different possible transitions, different polaron cyclotron masses can be defined and measured in the experiment, but only if the corresponding electron–phonon contribution to the energy splitting vanishes does the limit of $B \to 0$ contain the polaron mass. Level crossing between the states $|0, \pm 1; 0_q\rangle$ and $|0, 0; 1_q\rangle$, and consequently the existence of a resonant magnetopolaron arises either under the condition $\omega_L > \Omega$ at $\omega_c = (\omega_L^2 - \Omega^2)/\omega_L$ for the states $|0, +1; 0_q\rangle$ and $|0, 0; 1_q\rangle$ or $\omega_L < \Omega$ at $\omega_c = (\Omega^2 - \omega_L^2)/\omega_L$ for the states $|0, -1; 0_q\rangle$ and $|0, 0; 1_q\rangle$. This resonant cyclotron frequency ω_c depends on the confinement frequency Ω and can be much smaller than in the 2D and 3D case where the level crossing occurs at $\omega_c = \omega_L$. It is shown that the polaron cyclotron mass increases with reduced dimensionality of the magnetopolaron.

To improve on these results one has to include in the calculation the non-parabolicity of the conduction band (band-structure effect), the non-parabolicity of the confinement potential, deviations from the circularity of the QD, the finite width of the QD in the growth direction and, if many electrons are present, occupation and screening effects.

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